Adaptive approximation of high-dimensional functions with tree tensor networks for Uncertainty Quantification.

Uncertainty quantification problems for numerical models require a lot of simulations, often very computationally costly (in time and/or memory). This is why it is essential to build surrogate models that are cheaper to evaluate. In practice, the output of a numerical model is represented by a function, then the objective is to construct an approximation.

The aim of this thesis is to construct a controlled approximation of a function while using as few evaluations as possible.
In a first time, we propose a new method based on weighted least-squares to construct the approximation of a function onto a linear approximation space. We prove that the projection verifies a numerical stability property almost surely and a quasi-optimality property in expectation. In practice we observe that the sample size is closer to the dimension of the approximation space than with existing weighted least-squares methods.
For high-dimensional approximation, and in order to exploit potential low-rank structures of functions, we consider the model class of functions in tree-based tensor formats. These formats admit a multilinear parametrization with parameters forming a tree network of low-order tensors and are therefore also called tree tensor networks. In this thesis we propose an algorithm for approximating functions in tree-based tensor formats. It consists in constructing a hierarchy of nested subspaces associated to the different levels of the tree. The construction of these subspaces relies on principal component analysis extended to multivariate functions and the new weighted least-squares method. To reduce the number of evaluations necessary to build the approximation with a certain precision, we propose adaptive strategies for the control of the discretization error, the tree selection, the control of the ranks and the estimation of the principal components.

Keywords: high-dimensional approximation, low-rank approximation, tree-based tensor formats, principal component analysis, adaptive strategies,weighted least-squares.

Mots-clés : approximation en grande dimension, approximation de faible rang, formats de tenseurs basés sur des arbres, analyse en composantes principales, strat\'egies adaptatives, moindres carrés pondérés.